Nonlinear Unsteady Transonic Flow Theory—Local Linearization Solution for Two-Dimensional Flow

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Theme

ANALYSIS is described for the prediction of pressure distributions on aerodynamic configurations undergoing both rigid body and elastic oscillations. The theory is based on dividing the flow into steady and unsteady components and solving the resulting equations by the local linearization method. The analysis is developed generally, but specific applications are confined here to airfoils in two-dimensional flow with freestream Mach number $M_\infty = 1$. The results are shown to converge correctly to nonlinear quasi-steady theory as the reduced frequency of oscillation based on chord, $\bar{k} \to 0$, and to linear acoustic theory as $\bar{k}$ becomes large ($\bar{k} \geqq 2$). For $\bar{k} \leqq 0.1$, the results display significant nonlinear thickness effects coupled to the steady-state solution, and demonstrate the complete inadequacy of linear theory in this frequency regime of prime importance in many flutter applications.

Contents

It is known[1-3] that a major body of transonic flow problems can be treated accurately using the inviscid, nonlinear small-disturbance theory associated with the equation

$$(1-M_\infty{}^2)\phi_{xx}+\phi_{yy}+\phi_{zz} = M_\infty{}^2(\gamma+1)\phi_x\phi_{xx}+M_\infty{}^2\phi_{tt}+2M_\infty{}^2\phi_{xt} \tag{1}$$

where M_∞ is the freestream Mach number; x, y, z are nondimensional coordinates normalized by the wing chord c with the x-axis directed rearward and aligned with the wing chord, the y-axis in the plane of the wing, and the z-axis directed vertically upward; t is nondimensional time normalized by c/U_∞ where U_∞ is the freestream velocity; γ is the ratio of specific heats, equal to 7/5 for air; and ϕ is the dimensionless perturbation velocity potential. For steady or very slow (quasi-steady) motions, ϕ_{tt} and ϕ_{xt} can be disregarded, whereas for somewhat more rapid motions, it is adequate to retain ϕ_{xt} while neglecting ϕ_{tt}. If the unsteady motions are sufficiently rapid, Eq. (1) may be linearized by disregarding $\phi_x\phi_{xx}$ in the calculation of the unsteady component of the flow. Although many important applications remain to be worked out for these special cases, we have considered the more general case given by Eq. (1), encompassing all frequencies in one analysis.

For general oscillatory motions, it is convenient to expand the solution into a steady and unsteady component. Thus, we may set

$$\phi(x,y,z,t) = \phi_1(x,y,z)+R.P.[\tilde{\phi}(x,y,z)e^{i\bar{k}t}] \tag{2}$$

in which ϕ_1 is the steady perturbation potential that satisfies Eq. (1) without the unsteady terms, $\tilde{\phi}$ is the complex amplitude of the oscillatory perturbation velocity potential and $\bar{k} = \omega c/U_\infty$. Substitution into Eq. (1) and omission of the nonlinear term $\tilde{\phi}_x\tilde{\phi}_{xx}$ on the basis of a restriction to small amplitude oscillations yields

$$(1-M_\infty{}^2)\tilde{\phi}_{xx}+\tilde{\phi}_{yy}+\tilde{\phi}_{zz} = M_\infty{}^2(\gamma+1)(\phi_{1_x}\tilde{\phi}_{xx}+\phi_{1_{xx}}\tilde{\phi}_x)+2iM_\infty{}^2\bar{k}\tilde{\phi}_x-M_\infty{}^2\bar{k}^2\tilde{\phi} \tag{3}$$

which, although linear, remains quite formidable to solve because of the variable coefficients and mixed elliptic-hyperbolic type.

Correspondingly, the boundary condition at the wing is decomposed into components associated with the following expression for the normalized coordinates $Z(x,y,t)$ of the upper surface of a thin symmetrical wing undergoing either rigid body or elastic oscillations.

$$Z(x,y,t) = \tau\bar{Z}(x,y)+R.P.[\delta\tilde{Z}(x,y)e^{i\bar{k}t}] \tag{4}$$

where $(\bar{Z},\tilde{Z})$ are normalized functions describing the steady and oscillatory components of the wing ordinates, and (τ,δ) are the thickness ratio of the wing and the dimensionless amplitude of the unsteady oscillations. These considerations lead to the boundary conditions.

$$\phi_{1_z}(x,y,0\pm) = \pm\tau\,\partial\bar{Z}(x,y)/\partial x,\quad \tilde{\phi}_z(x,y,0\pm) = \delta[\partial\tilde{Z}(x,y)/\partial x+i\bar{k}\tilde{Z}(x,y)] \tag{5}$$

Similarly, the pressure coefficient is given by

$$C_p = C_{p_1}+R.P.(\tilde{C}_p e^{i\bar{k}t}) = -2\phi_{1_x}-R.P.[2(\tilde{\phi}_x+i\bar{k}\tilde{\phi})e^{i\bar{k}t}] \tag{6}$$

The problem has been solved approximately by application of the local linearization method developed originally for steady transonic flow studies.[4-6] The procedure requires first that the steady-state solution ϕ_1 be obtained to evaluate the variable coefficients of Eq. (3). Then, an approximate solution of Eq. (3) is determined by replacing temporarily the variable coefficients by constants, solving the simplified equation for $\tilde{\phi}(x,z)$, calculating the unsteady surface acceleration $\tilde{\phi}_{xx}(x,0)$, replacing the constant coefficients by the functions they originally represented, and finally integrating the resultant second ordinary differential equation to determine $\tilde{\phi}_x(x,0)$ and $\tilde{\phi}(x,0)$ to be used in Eq. (6). Full details of these steps may be found in the complete paper. Of the various results given there, we present those for flow with $M_\infty = 1$ past a parabolic-arc airfoil having ordinates given by $Z_1(x) = 2\tau(x-x^2)$ executing oscillatory motions of vertical translation ($\tilde{Z} = 1$) or of pitch about the nose ($\tilde{Z} = x$).

Figure 1 exhibits the solutions for the normalized magnitude and phase (in degrees) of the unsteady surface pressure distributions $\tilde{C}_p$ on the upper surface of a 6% thick parabolic-arc airfoil oscillating in vertical translation (plunging) at various $\bar{k}$. The results for the magnitude have been normalized by $\delta\bar{k}$, rather than the usual δ because the solution contains a linear factor in $\bar{k}$ for this case. Also indicated are the results provided

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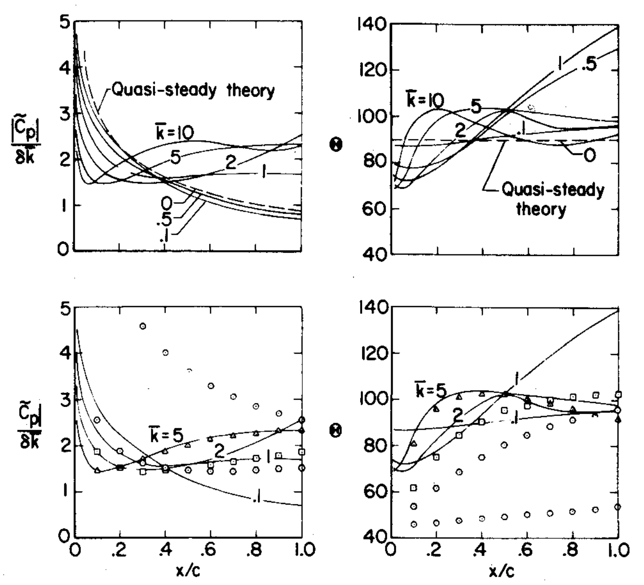

Fig. 1 Unsteady pressure distributions on a 6% thick parabolic-arc airfoil in plunging oscillation; local linearization ———, quasi-steady - - - - - - -, linear theory $\bar{k} = 0.1$, ⊙; $\bar{k} = 1.0$, ◇; $\bar{k} = 2.0$, ⊡; $\bar{k} = 5.0$, △.

by quasi-steady theory and by linear acoustic theory to which the present results converge for small and large $\bar{k}$. The close correspondence between the nonlinear and the quasi-steady results for $\bar{k} = 0.1$ implies that the nonlinear thickness effects of the steady flow exert a primary influence on the unsteady flow for very low frequency oscillations, and indicates that quasi-steady theory can provide good results in this range. On the other hand, the comparisons with linear theory show that approximation to be totally useless for small $\bar{k}$, and only begin to approach the nonlinear results reasonably when $\bar{k}$ is about 2 for the magnitude and about 5 for the phase.

Figure 2 shows the analogous results for the same airfoil oscillating in pitch about its nose. These results also display a close correspondence with quasi-steady theory for $\bar{k} = 0.1$, and a smooth transition to the linear acoustic results for large $\bar{k}$. In between exists a substantial frequency range that requires use of the nonlinear theory for an adequate representation.

To demonstrate explicitly the importance of nonlinear thickness effects, Fig. 3 is presented to show, for three fixed values of $\bar{k}$, the magnitude and phase of $\tilde{C}_p$ due to plunging oscillations of parabolic-arc airfoils of various τ. Clearly evident in the results for $\bar{k} = 0.1$ are the enormous changes as τ increases only slightly from zero (for which the present theory and linear acoustic theory are the same) to 0.01. These results demonstrate both the importance of thickness effects and the inadequacy of linear theory at low frequencies, even for exceptionally thin airfoils. The results for $\bar{k} = 1$ show that variations of τ have much smaller effects on the results than at $\bar{k} = 0.1$ and also are largely restricted to the aft portion of the airfoil. Although the phase angles, which are generally more sensitive to change than the magnitude, remain somewhat separated for each τ, the trend for all the curves to move toward the zero thickness (linear) result is clear. The trend is notably evident for $\bar{k} = 10$, for which there is essentially no difference in the results from those of linear theory, except near the tail where presumably the flow finally has had enough time to react to some of the nonlinear effects of thickness in the steady flow.

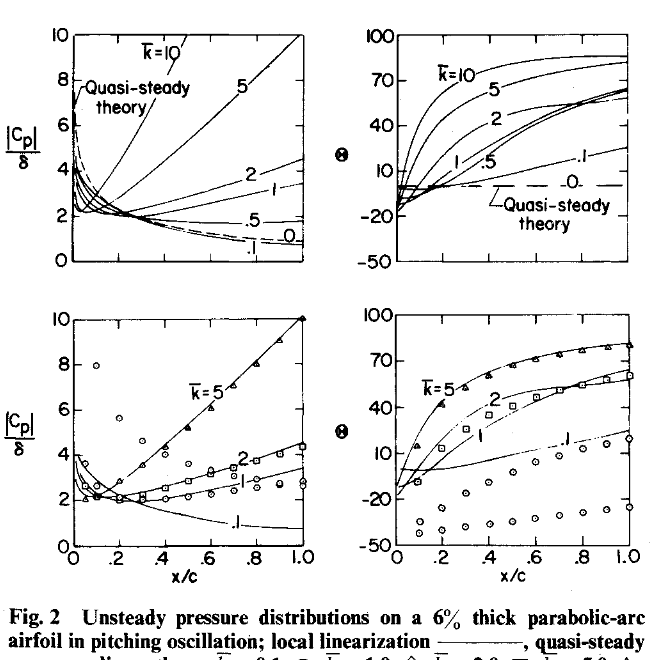

Fig. 2 Unsteady pressure distributions on a 6% thick parabolic-arc airfoil in pitching oscillation; local linearization ———, quasi-steady - - - - - - -, linear theory $\bar{k} = 0.1$, ⊙; $\bar{k} = 1.0$, ◇; $\bar{k} = 2.0$, ⊡; $\bar{k} = 5.0$, △.

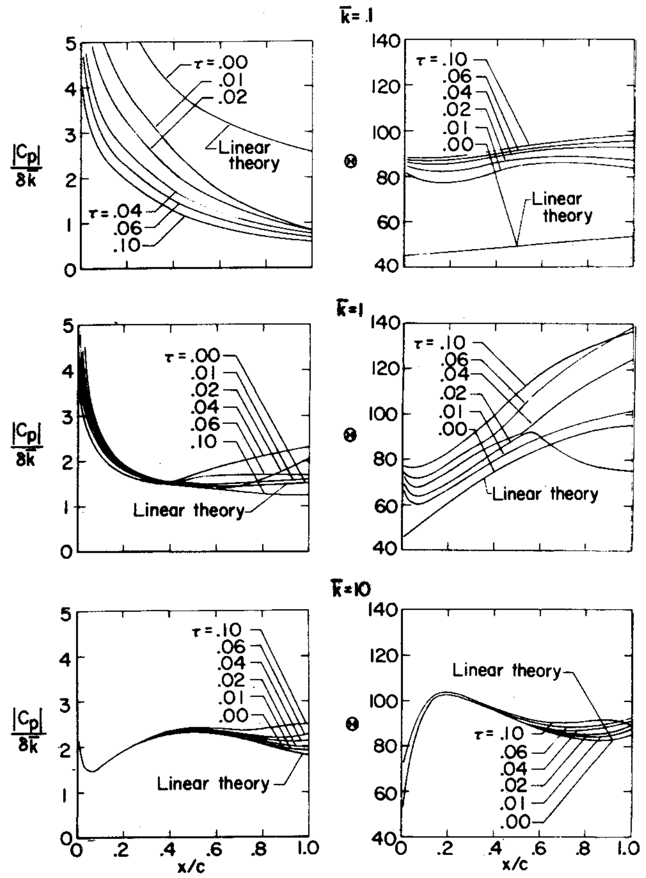

Fig. 3 Unsteady pressure distributions on various thickness ratio parabolic-arc airfoils in plunging oscillation at $\bar{k} = 0.1, 1.0$, and 10.

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